

JEE Main Home Practice Test - 7 | JEE - 2024

Date: 06/01/2024

Maximum Marks: 300

Timing: 10:00 AM to 1:00 PM

Duration : 3.0 Hours

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300**.
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2** contains **10 Numerical Value Type Questions** Out of which **ONLY 5 (any)** questions have to be attempted. You will **NOT** be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.
The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.
6. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

Syllabus

Physics: Full Syllabus

Chemistry: Full Syllabus

Mathematics: Full Syllabus

Name of the Candidate (In CAPITALS) :

Roll Number :

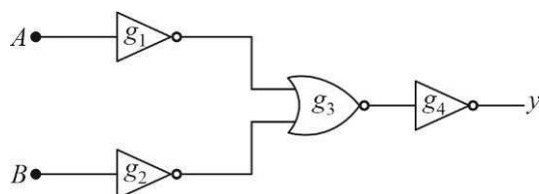
OMR Bar Code Number :

Candidate's Signature : Invigilator's Signature

PART - I : PHYSICS**100 MARKS****SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE CHOICE is correct.

- The pitch of a screw gauge is 1 mm and there are 100 divisions on circular scale. When faces A and B are just touching each other without putting anything between the studs 32^{nd} division of the circular scale coincides with the reference line. When a glass plate is placed between the studs, the linear scale reads 4 divisions and the circular scale reads 16 divisions. Zero of linear scale is not hidden from circular scale when A and B touches each other. Thickness of the glass plate is:
 (A) 1.55 mm (B) 2.54 mm (C) 3.84 mm (D) 5.64 mm
- At what temperature is the rms speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at -20°C ? (Atomic mass of $\text{Ar} = 39.9\text{ u}$, of $\text{He} = 4.0\text{ u}$).
 (A) $5 \times 10^3\text{ K}$ (B) $3.2 \times 10^3\text{ K}$ (C) $3 \times 10^3\text{ K}$ (D) $2.52 \times 10^3\text{ K}$
- The combination of gates below is equivalent to:

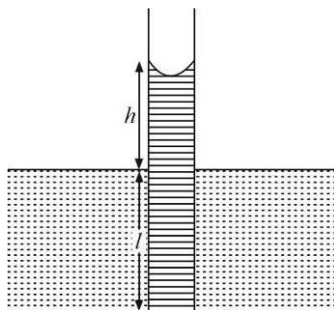


- (A) AND gate (B) XOR gate (C) NOR gate (D) NAND gate
- An electric appliance supplies 12000 J/min heat to a thermodynamic system. If the system delivers a power of 100 W . How long it would take to increase the internal energy by $2.5 \times 10^3\text{ J}$?
 (A) 30 sec (B) 20 sec (C) 25 sec (D) 40 sec

Space for Rought Work

5. A hydrogen like atom of atomic number Z is in an excited state quantum number $2n$. It can emit a maximum energy photon of 204 eV . If it makes a transition to quantum state n , a photon of energy 40.8 eV is emitted. The value of n will be:
- (A) 1 (B) 2 (C) 3 (D) 4

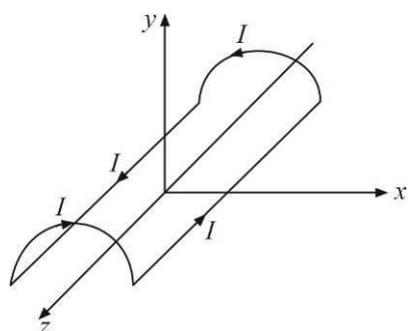
6. Water rises to a height h in a capillary tube lowered vertically into water to a depth l as shown in the figure. The lower end of the tube is now closed and the tube is then taken out of the water and opened again. The length of the water column remaining in the tube will be:



- (A) $2h$ if $l \geq h$ and $l + h$ if $l \leq h$ (B) h if $l \geq h$ and $l + h$ if $l \leq h$
- (C) $4h$ if $l \geq h$ and $l - h$ if $l \leq h$ (D) $\frac{h}{2}$ if $l \geq h$ and $l + h$ if $l \leq h$
7. A photosensitive metallic surface has work function $h\nu_0$. If photons of energy $2h\nu_0$ fall on this surface the electrons come out with a maximum velocity of $4 \times 10^6\text{ m/s}$. When the photon energy is increased to $5h\nu_0$, then maximum velocity of photo electron will be:
- (A) $2 \times 10^6\text{ m/s}$ (B) $2 \times 10^7\text{ m/s}$ (C) $8 \times 10^5\text{ m/s}$ (D) $8 \times 10^6\text{ m/s}$

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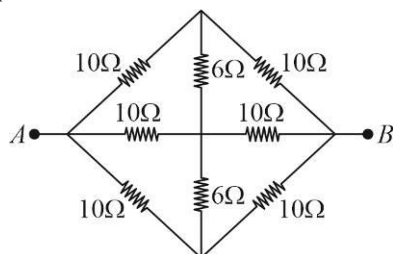
8. A gun of mass m_1 fires a bullet of mass m_2 with a horizontal speed v_0 . The gun is fitted with a concave mirror of focal length f facing towards a receding bullet. Speed of separations of the bullet and the image just after the gun was fired will be:
- (A) $3\left[1 + \frac{m_2}{m_1}\right]v_0$ (B) $\left[1 + \frac{m_2}{m_1}\right]v_0$ (C) $2\left[1 + \frac{m_2}{m_1}\right]v_0$ (D) $4\left[1 + \frac{m_2}{m_1}\right]v_0$
9. An inductor coil stores 32 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8 A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds:
- (A) 0.5 (B) 0.35 (C) 0.9 (D) 0.1
10. An inductor of 2 H , capacitance $18\mu\text{F}$ and a resistance of $10\text{ k}\Omega$ are connected in series to an AC source of 20 V with adjustable frequency. At what frequency current in circuit will be maximum?
- (A) $\frac{500}{3\pi}\text{ Hz}$ (B) $\frac{250}{3\pi}\text{ Hz}$ (C) $\frac{750}{3\pi}\text{ Hz}$ (D) $\frac{150}{3\pi}\text{ Hz}$
11. We have a uniform current carrying wire loop bent in the form of a semicircular cylinder, as shown in the figure. The magnetic field at the origin is:



- (A) Directed along x-axis (B) Directed along y-axis
(C) Directed along z-axis (D) Zero

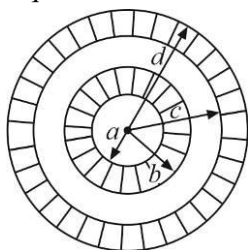
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12. Find equivalent resistance between A and B .

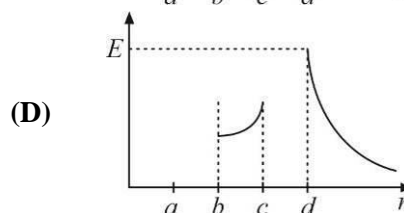
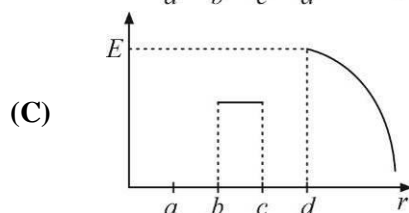
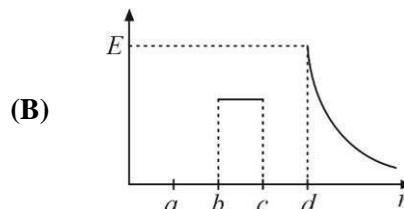
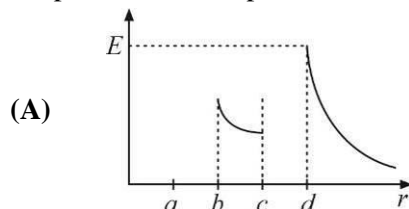


- (A) $\frac{10}{3}\Omega$ (B) $\frac{20}{3}\Omega$ (C) $\frac{5}{3}\Omega$ (D) $\frac{40}{3}\Omega$

13. There are two conducting hollow spherical shells. One has inner radius a and outer radius b . Other has inner radius c and outer radius d . Inner shell (of radius a and b) has total charge $+2q$, and outer shell has charge $+4q$.



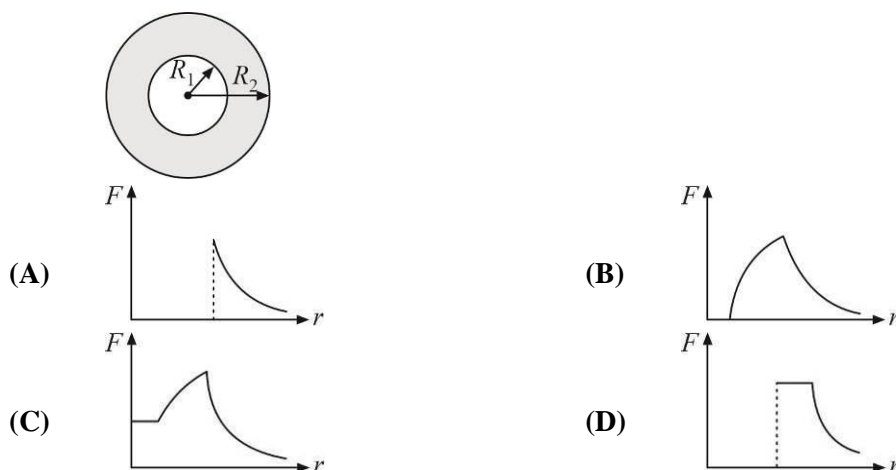
Graph of radial component of E as a function of r will be:



Space for Rought Work

14. An isolated parallel plate capacitor has circular plates of radius 4.0 cm . The gap is filled with a partially conducting material of dielectric constant K and conductivity $5.0 \times 10^{-14} \Omega^{-1} m^{-1}$. When the capacitor is charged to a surface charge density of $15 \mu C/cm^2$, the initial current between the plates is $1.0 \mu A$. If the total joule heating produced is 7500 J , determine the separation of the capacitor plates.
- (A) 10 mm (B) 5 mm (C) 15 mm (D) 20 mm

15. Sphere of mass M and radius R_2 has a concentric cavity of radius R_1 as shown in the figure. The force F exerted by the sphere on a particle of mass m located at a distance r from the centre of sphere varies as ($0 \leq r \leq \infty$).



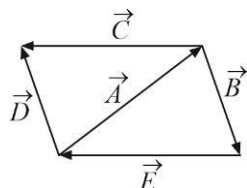
16. If a solid will break under compressive stress greater than 13 atm and that solid has a specific gravity of 4 , what is the maximum height of a cylindrical column made from the solid that can be built at the earth's surface? ($1\text{ atm} = 10^5\text{ Pa}$)
- (A) $4m$ (B) $32.5m$ (C) $24m$ (D) $30m$

Space for Rought Work

17. At points A, B, C on a straight line segment we fix charge Q, 2Q and 4Q connecting them to each other by inextensible threads. The lengths of segments $AB = BC = L$, $\frac{kQ^2}{L^2} = T$. The force of tension in the sections AB and BC are respectively:

(A) 2T, 4T (B) 3T, 9T (C) T, 5T (D) T, 4T

18. Choose correct option:



- (A) $\vec{A} + \vec{B} + \vec{E} = 0$ (B) $\vec{C} - \vec{D} = -\vec{A}$
 (C) $\vec{B} + \vec{E} - \vec{C} = -\vec{D}$ (D) All of the above

19. **Statement I :** The Zener diode work on the principle of breakdown voltage.

Statement II : In Zener diode current increases suddenly after breakdown voltage.

- (A) Both statement I and statement II are true
 (B) Both statement I and statement II are false
 (C) Statement I is true but statement II is false
 (D) Statement I is false but statement II is true

20. If velocity v , acceleration A and force F are chosen as fundamental quantities, then the dimensional formula of angular momentum in terms of v , A and F would be:

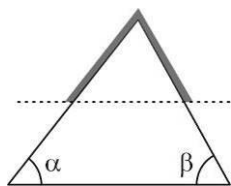
- (A) $FA^{-1}v$ (B) Fv^3A^{-2} (C) Fv^2A^{-1} (D) $F^2v^2A^{-1}$

Space for Rought Work

SECTION-2

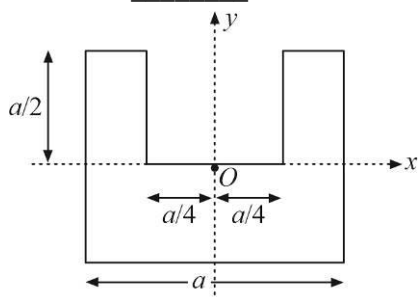
Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

1. A uniform rope of length L and mass M is placed on a smooth fixed wedge as shown. Both ends of rope are at same horizontal level. The rope is initially released from rest, then the magnitude of initial acceleration is rope is _____ m/s^2 . ($\alpha = 53^\circ$, $\beta = 60^\circ$)



2. A square plate of edge $a/2$ is cut out from a uniform square plate of edge ' a ' as shown in figure. The mass of the remaining portion is M . The moment of inertia of the remaining portion about an axis passing through ' O ' (centre of the square of side a) and perpendicular to plane of the plate is $x \frac{Ma^2}{16}$.

Value of x will be _____.



Space for Rought Work

3. Two separate air bubbles (radii $0.004m$ and $0.002m$) formed of the same liquid (surface tension $0.07 N/m$) come together to form a double bubble. Radius of curvature of the internal film surface common to both the bubbles is $x \times 10^{-3}m$. Value of x is _____.
4. A standing wave pattern is formed on a string. One of the waves is given by equation $y_1 = a \cos\left(\omega t - kx + \frac{\pi}{3}\right)$, then the equation of the other wave such that at $x = 0$ a node is formed is $y_2 = a \cos\left[\omega t + kx + \frac{P\pi}{3}\right]$. Value of P will be _____.
5. A bullet of mass $20 g$ has an initial speed of $1 ms^{-1}$, just before it starts penetrating a mud wall of thickness $20 cm$. If the wall offers a mean resistance of $5 \times 10^{-2} N$, the speed of the bullet after emerging from the other side of the wall is close to _____ ms^{-1}
6. The incident intensity on a horizontal surface at sea level from sun corresponding to all radiations of EM wave is about $1 kW/m^2$. Assuming that 50% of this intensity is reflected and 50% is absorbed. The radiation pressure on this horizontal surface is $x \times 10^{-6} Pa$. Value of x is _____.
-

Space for Rought Work

7. In a Young's double slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelength $\lambda = 12000\text{\AA}$ and $\lambda' = 10000\text{\AA}$. Minimum distance from the common central bright fringe on a screen 2 m from the slits where a bright fringe from one interference pattern coincide with a bright fringe from the other will be _____(mm).
8. A string of length 4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N . Identical wave pulses are produced at one end at equal intervals of time Δt . The minimum value of Δt which allows constructive interference between successive pulses is _____.
9. A man drops a ball downside from the roof of a tower of height 400 meters . At the same time another ball is thrown upside with a velocity 50 meter/sec from the surface of the tower, then they will meet at some height from the surface of the tower. Value of height from surface will be $10x\text{ metre}$. Value of x will be _____.
10. A satellite is revolving round the earth in a circular orbit of radius ' a ' with velocity v_0 . A particle is projected from satellite in a forward direction with relative velocity $v = \left(\sqrt{\frac{5}{4}} - 1\right)v_0$. Then it is found that the maximum distance of particle from earth's centre is $\frac{na}{3}$. Then the value of n is _____.
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Space for Rought Work

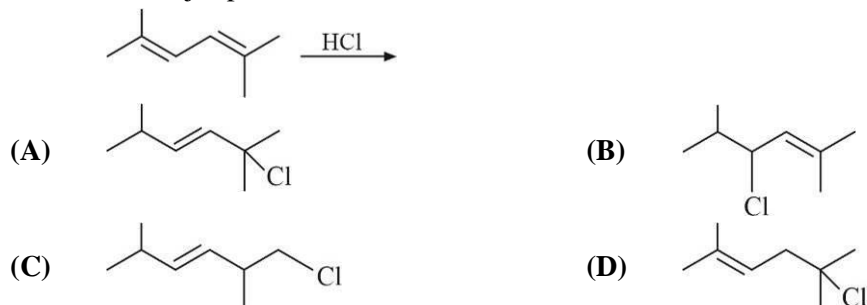
PART - II : CHEMISTRY**100 MARKS****SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

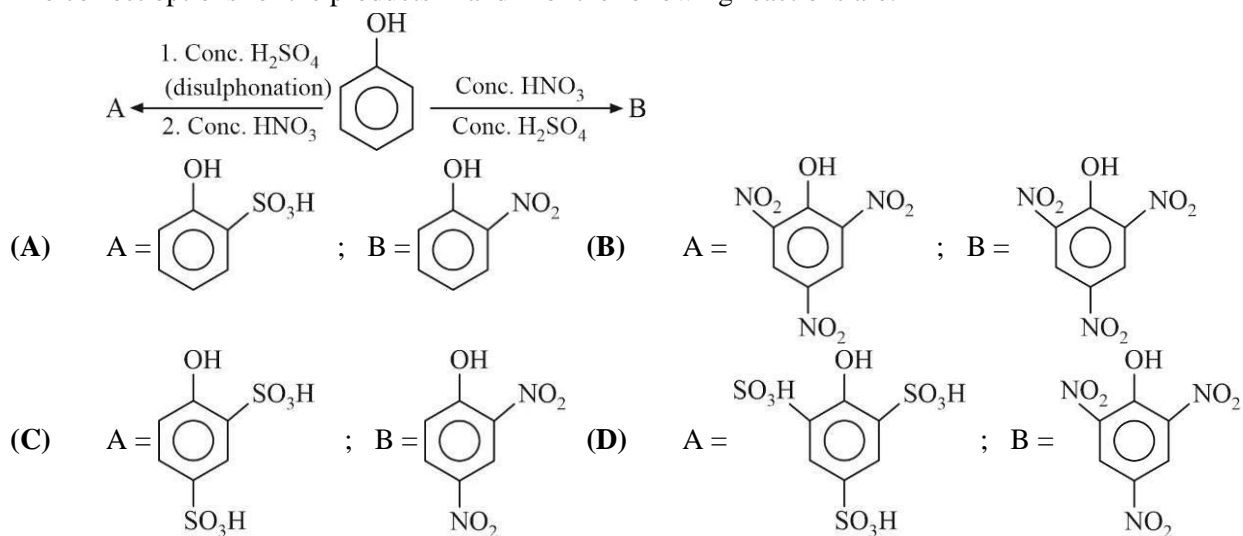
1. The correct statement is:

- (A) I_2 is more reactive than ICl (B) On hydrolysis ICl forms $HOCl$ and HI
 (C) Cl_2 is more reactive than ICl (D) On hydrolysis ICl gives HOI and HCl

2. What is the major product in the reaction shown?



3. The correct options for the products A and B of the following reactions are:



Space for Rought Work

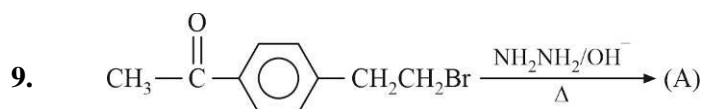
4. Given below are two statements:
Statement I: According to Bohr's model of an atom, qualitatively the magnitude of radius of orbit increases with decrease in positive charge on the nucleus as there is lesser hold on the electron by the nucleus.
Statement II: According to Bohr's model of an atom, qualitatively the magnitude of radius of orbit increases with decrease in principal quantum number.
 In the light of the above statements, choose the most appropriate answer from the options given below:
 (A) Both statement I and statement II are true
 (B) Both statement I and statement II are false
 (C) Statement I is false but statement II is true
 (D) Statement I is true but statement II is false
5. Which one of the following is not applicable to the phenomenon of adsorption?
 (A) $\Delta H > 0$ (B) $\Delta G < 0$ (C) $\Delta S < 0$ (D) $\Delta H < 0$
6. A solid compound 'X' on heating gives CO_2 gas and a residue. The residue, when mixed with water forms 'Y'. On passing an excess of CO_2 through 'Y' in water, a clear solution 'Z' is obtained. On boiling 'Z', compound 'X' is reformed. The compound 'X' is:
 (A) $\text{Ca}(\text{HCO}_3)_2$ (B) CaCO_3 (C) Na_2CO_3 (D) K_2CO_3
7. Generally transition elements form coloured salts due to the presence of unpaired electrons. Which of the following compounds will be coloured in solid state?
 (A) Ag_2SO_4 (B) CuF_2 (C) ZnF_2 (D) Cu_2Cl_2
8. Excess of 2-Methylbutane on reaction with Br_2 in presence of light at 125°C gives which one of the following, as the major product?
- (A) $\text{BrCH}_2 - \overset{\text{CH}_3}{\underset{|}{\text{CH}}} - \text{CH}_2\text{CH}_3$

(C) $\text{CH}_3 - \overset{\text{CH}_3}{\underset{|}{\text{CH}}} - \underset{\text{Br}}{\underset{|}{\text{CH}}} - \text{CH}_3$

(B) $\text{CH}_3 - \overset{\text{CH}_3}{\underset{\text{Br}}{\underset{\text{CH}_3}{|}{\text{C}}}} - \text{CH}_2\text{CH}_3$

(D) $\text{CH}_3 - \underset{\text{Br}}{\underset{|}{\text{CH}}} - \text{CH}_2\text{CH}_2\text{Br}$

Space for Rought Work



The product (A) in the above reaction is:

- (A) $\text{CH}_3\text{CH}_2-\text{C}_6\text{H}_4-\text{CH}_2\text{CH}_2\text{OH}$ (B) $\text{CH}_3\text{CH}_2-\text{C}_6\text{H}_4-\text{CH}=\text{CH}_2$
 (C) $\text{CH}_3\text{CH}_2-\text{C}_6\text{H}_4-\text{CH}_2\text{CH}_2\text{Br}$ (D) $\text{CH}_3\text{CH}_2-\text{C}_6\text{H}_4-\text{CH}(\text{Br})\text{CH}_3$

10. Read the assertion and reason carefully to mark the correct option out of the options given below:

Assertion : Formaldehyde cannot be prepared by Rosenmund's reduction.

Reason : Acid chlorides can be reduced into aldehydes with hydrogen in boiling xylene using palladium or platinum as a catalyst supported on barium sulphate. This is known as Rosenmund's reduction.

- (A) If both assertion and reason are true and the reason is the correct explanation of the assertion
 (B) If both assertion and reason are true but reason is not the correct explanation of the assertion
 (C) If assertion is true but reason is false
 (D) If the assertion and reason both are false
11. Which of the following is violet coloured complex?

- (A) $[\text{Cu}(\text{H}_2\text{O})_4]^{2+}$ (B) $[\text{Co}(\text{NH}_3)_6]^{3+}$
 (C) $[\text{CoCl}(\text{NH}_3)_5]^{2+}$ (D) $[\text{Co}(\text{CN})_6]^{3-}$

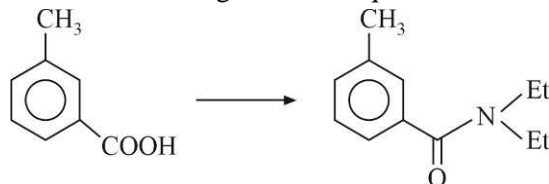
Space for Rought Work

12. Given $\varepsilon_{\text{Cr}^{+3}/\text{Cr}}^0 = -0.74\text{V}$; $\varepsilon_{\text{MnO}_4^-/\text{Mn}^{2+}}^0 = 1.51\text{V}$

$$\varepsilon_{\text{Cr}_2\text{O}_7^{2-}/\text{Cr}^{+3}}^0 = 1.33\text{V}; \varepsilon_{\text{Cl}_2/\text{Cl}^-}^0 = 1.36\text{V}$$

Based on the data given above, the strongest oxidizing agent will be:

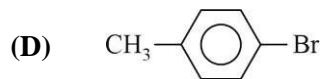
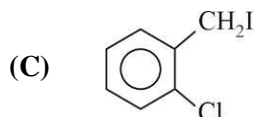
- (A) Cl_2 (B) Cr^{+3} (C) Mn^{+2} (D) MnO_4^-
13. The active ingredient of Odomos Cream used to protect ourselves from mosquito bites is m-diethyltoluamide. Which of the following reaction sequence will best accomplish this transformation?



- (A) $\xrightarrow{\text{LiNEt}_2}$ (B) $\xrightarrow{\text{LiAlH}_4} \xrightarrow{(\text{Et})_2\text{NH}}$
- (C) $\xrightarrow{\text{NH}_3} \xrightarrow{\text{CH}_3\text{F (excess)}}$ (D) $\xrightarrow{\text{SOCl}_2} \xrightarrow{\text{Et}_2\text{NH}}$
14. The conversion of hydroxyapatite to fluorapatite occurs in presence of water. The correct formula of fluorapatite is:
- (A) $(\text{Ca}_3(\text{PO}_4)_2 \cdot \text{CaF}_2)$ (B) $(3\text{Ca}_3(\text{PO}_4)_2 \cdot \text{CaF}_2)$
- (C) $[3\text{Ca}_3(\text{PO}_4)_2 \cdot \text{Ca}(\text{OH})_2]$ (D) $[3\text{Ca}(\text{OH})_2 \cdot \text{CaF}_2]$

Space for Rought Work

15. Which of the following will give yellow ppt. on shaking with an aqueous solution of NaOH followed by acidification with dil. HNO_3 and addition of AgNO_3 solution?



16. Given below are two statements:

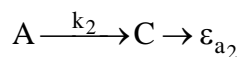
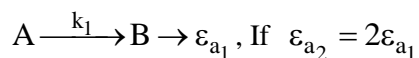
Statement I: In the titration between weak acid and strong base methyl orange is suitable as an indicator.

Statement II: For titration of acetic acid with NaOH phenolphthalein is a suitable indicator.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (A) Both statement I and statement II are false
 (B) Both statement I and statement II are true
 (C) Statement I is true but statement II is false
 (D) Statement I is false but statement II is true

17. A reaction (A) forms two products:



Then k_1 & k_2 are related as

- (A) $k_2 = k_1 e^{\varepsilon_a/RT}$ (B) $k_2 = k_1 e^{\varepsilon_{a_2}/RT}$
 (C) $k_2 = k_1 e^{-\varepsilon_{a_1}/RT}$ (D) $k_1 = A \quad k_1 = A k_2 e^{\varepsilon_{a_2}/RT}$

Space for Rought Work

18. Given below are two statements:

Statement I: Molar conductivity of all electrolytes varies linearly with dilution.

Statement II: Molar conductivity increases with decrease in concentration of electrolyte.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (A) Statement I is true but statement II is false
- (B) Both statement I and statement II are false
- (C) Statement I is false but statement II is true
- (D) Both statement I and statement II are true

19. The correct sequential addition of reagents in the preparation of 4-nitro benzaldehyde from benzene is:

- (A) Conc.HNO₃ + conc.H₂SO₄, CH₃Cl / Anhydrous AlCl₃, Cl₂, NaOH
- (B) Cl₂, CH₃Cl / Anhydrous AlCl₃, conc. HNO₃ + conc. H₂SO₄, NaOH
- (C) CH₃Cl / Anhydrous AlCl₃, Conc. HNO₃ + conc. H₂SO₄, Cl₂, NaOH
- (D) CH₃Cl / Anhydrous AlCl₃, Cl₂, conc. HNO₃ + conc H₂SO₄, NaOH

20. Read the assertion and reason carefully to mark the correct option out of the options given below:

Assertion : Lower aldehyde and ketones are soluble in water but the solubility decreases as molecular mass increases.

Reason : Aldehydes and ketones can be distinguished by Tollen's reagent.

- (A) If both assertion and reason are true and the reason is the correct explanation of the assertion
- (B) If both assertion and reason are true but reason is not the correct explanation of the assertion
- (C) If assertion is true but reason is false
- (D) If the assertion and reason both are false

Space for Rought Work

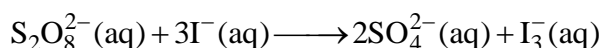
SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

1. Of the following four aqueous solutions, total number of those solutions whose freezing point is higher than that of 0.10 M $\text{Al}_2(\text{SO}_4)_3$ is _____. (Integer answer)
(i) 0.10 M $\text{Ba}_3(\text{PO}_4)_2$ (ii) 0.10 M Na_2SO_4
(iii) 0.10 M KCl (iv) 0.10 M Li_3PO_4
2. The total number of positive charge in the tetrapeptide, Gly-Glu-Asp-Tyr, at pH 6 will be _____. (Integer answer)
3. The concentration of hydroxide ion $[\text{OH}^-]$ in a solution containing 0.35 mol of $(\text{CH}_3)_3\text{N}$, 0.05 mol of $(\text{CH}_3)_3\text{NH}^+$, and sufficient water to make up one litre of solution, (K_b (trimethyl amine) $= 6.3 \times 10^{-5}$) is 4.41×10^{-x} . The value of x is [given $K_w = 1 \times 10^{-14}$]. (Nearest integer)
4. You are given the following data:
 $\Delta_{\text{sub}}H^\circ (\text{Na}) = 109 \text{ kJ mol}^{-1}$, $\Delta_{\text{dissociation}}H^\circ (\text{Cl}_2) = 244 \text{ kJ mol}^{-1}$
 $\Delta_{\text{ionisation}}H^\circ (\text{Na}) = 496 \text{ kJ mol}^{-1}$, $\Delta_{\text{eg}}H^\circ (\text{Cl}) = -349 \text{ kJ mol}^{-1}$,
 $\Delta_{\text{f}}H^\circ (\text{NaCl}) = -410 \text{ kJ mol}^{-1}$.
The magnitude of lattice enthalpy of NaCl (s) will be:
5. The ratio of number of water molecules in Epsom salt and Glauber's salt is _____ $\times 10^{-1}$.

Space for Rought Work

6. The number of d electrons in the ground state electronic configuration of Gd^{3+} is _____.
[Atomic number of Gd = 64]
7. AB_2 is an polyhalide linear anion. The number of lone pairs of electrons on A is (Integer answer).
8. These are physical properties of an element belonging to Group 17:
 (A) Sublimation enthalpy (B) Ionisation enthalpy
 (C) Hydration enthalpy (D) Electron gain enthalpy
 (E) Bond dissociation energy
 The total number of above properties that affect the reduction potential is _____. (Integer answer)
9. The molality and molarity of a sulphuric acid solution are 94.5 and 11.5 respectively. The density of the solution is:
10. The following data was obtained for chemical reaction given below:



With respect to each reactant:

Exp. No.	Initial $[\text{S}_2\text{O}_8^{2-}]$ mol/L	Initial $[\text{I}^-]$ mol/L	Initial Rate ($\text{mol L}^{-1} \text{s}^{-1}$)
1	0.15	0.21	1.16
2	0.22	0.21	1.70
3	0.22	0.12	0.98

The order of the reaction with respect to $\text{S}_2\text{O}_8^{2-}$ is _____. [Integer answer]

Space for Rought Work

PART - III : MATHEMATICS**100 MARKS****SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. Let $P = \begin{bmatrix} \frac{\sqrt{3}+1}{2\sqrt{2}} & \frac{\sqrt{3}-1}{2\sqrt{2}} \\ \frac{1-\sqrt{3}}{2\sqrt{2}} & \frac{\sqrt{3}+1}{2\sqrt{2}} \end{bmatrix}$, $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, and $Q = P^T A P$, then $PQ^{2010}P^T$ is equal to:
- (A) $\begin{bmatrix} 2010 & 0 \\ 0 & 2010 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 2010 \\ 1 & 1 \end{bmatrix}$
- (C) $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
2. The coordinates of the point on the parabola $y^2 = 8x$, which is at minimum distance from the circle $x^2 + (y+6)^2 = 1$ are:
- (A) $(2, -4)$ (B) $(18, -12)$ (C) $(2, 4)$ (D) $(0, 0)$
3. Let a_1, a_2, a_3, \dots be an A.P. with $a_6 = 2$. Then the common difference of this A.P., which maximizes the product $a_1 a_4 a_5$, is:
- (A) $\frac{6}{5}$ (B) $\frac{2}{3}$ (C) $\frac{8}{5}$ (D) $\frac{3}{2}$

Space for Rought Work

4. Locus of z if $\arg[z - (1 + i)] = \begin{cases} \frac{3\pi}{4} & \text{when } |z| \leq |z - 2| \\ -\frac{\pi}{4} & \text{when } |z| > |z - 2| \end{cases}$ is:
- (A) Straight lines passing through (2, 0) (B) Lines passing through (2, 0) and (1, 1)
 (C) A line segment (D) A set of two rays
5. The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point (2, 3, -4) and parallel to the vector, $6\hat{i} + 3\hat{j} - 4\hat{k}$ is:
- (A) 6 (B) $4\sqrt{3}$ (C) $2\sqrt{13}$ (D) 7
6. A variable point $P(-x, -y, -z)$ defined such that $2x = -a - b - 1$, $y = -a - 4b - 2$ and $4z = 3a - 6b$ where a and b are fixed. If the minimum distance of P from $Q(1, 1, 1)$ is s the $|s|$ is:
- (A) 1 (B) 2 (C) 3 (D) 4
7. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is:
- (A) (5, 0) (B) $\left(-\frac{5}{3}, 0\right)$ (C) (-5, 0) (D) $\left(\frac{5}{3}, 0\right)$
8. For any two independent events E_1 and E_2 , $P\{(E_1 \cup E_2) \cap (\bar{E}_1 \cap \bar{E}_2)\}$ is:
- (A) Less than $\frac{1}{4}$ (B) Greater than $\frac{1}{4}$
 (C) Greater than or equal to $\frac{1}{2}$ (D) Greater than $\frac{1}{8}$

Space for Rought Work

9. The mean of six observations is 7 and their variance is $\frac{25}{3}$. If four observations are 5, 6, 8, 9, then the median of all the observations can be:
- (A) 7 (B) 6 (C) 11 (D) $\frac{11}{2}$
10. The solution of differential equation $2x^3y \, dy + (1 - y^2)(x^2y^2 + y^2 - 1)dx = 0$ is:
- (A) $x^2y^2 = (cx + 1)(1 - y^2)$ (B) $x^2y^2 = (cx + 1)(1 + y^2)$
(C) $x^2y^2 = (cx - 1)(1 - y^2)$ (D) None of these
11. $\int_0^\infty \frac{dx}{\left(x + \sqrt{x^2 + 1}\right)^3}$ is equal to:
- (A) $\frac{3}{8}$ (B) $\frac{1}{8}$ (C) $-\frac{3}{8}$ (D) None of these
12. If the system of equation $\lambda x + (b - a)y + (c - a)z = 0$, $(a - b)x + \lambda y + (c - b)z = 0$ and $(a - c)x + (b - c)y + \lambda z = 0$ has a non-trivial solution, then the value of λ is:
- (A) $\lambda = 0$ (B) $\lambda = 1$ (C) $\lambda = -1$ (D) None of these
13. The number of solutions for the equation $x^3 + 2x^2 + 5x + 2\cos x = 0$ in $[0, 2\pi]$ is:
- (A) 0 (B) 1 (C) 2 (D) 3
14. If $\lim_{x \rightarrow 1} \frac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to:
- (A) -4 (B) 1 (C) -7 (D) 5

Space for Rought Work

15. $A_r; 1, 2, 3, \dots, n$ are n points on the parabola $y^2 = 4x$ in the first quadrant. If $A_r \equiv (x_r, y_r)$ where x_1, x_2, \dots, x_n are in GP and $x_1 = 1, x_2 = 2$ then y_n is equal to:
- (A) $2^{\frac{n-1}{2}}$ (B) 2^{n+1} (C) $(\sqrt{2})^{n+1}$ (D) None of these
16. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is:
- (A) 5 (B) 6 (C) 7 (D) 8
17. If $f(x) = \sin \left(2 \tan^{-1} \cos \left(\tan^{-1} \left(\sqrt{\frac{1-x}{x}} \right) \right) \right)$, $0 < x < 1$, then:
- (A) $f(1) + f'(1) = 0$ (B) $f(1) + f'(1) = 1$
 (C) $f(1) + f'(1) = 2$ (D) $f(1) + f'(1) = 3$
18. If $P(A) = 3K, P(B) = K$, where 'A' and 'B' are two independent events and $P(\text{both } A \text{ and } B \text{ occur}) = 1/3$, then $P(\text{exactly one of } A, B \text{ occurs})$ is:
- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{9}$ (D) $\frac{2}{9}$
19. If ${}^{30}C_r$ is the coefficient of x^r in the expansion of $(1+x)^{30}$, then the value of $\sum_{r=1}^{30} r^2 {}^{30}C_r$ is equal to:
- (A) 465×2^{29} (B) 930×2^{29} (C) 465×2^{30} (D) 930×2^{30}
20. If $\vec{a} \times \vec{c} = \vec{b}$, $\vec{a} \cdot \vec{c} = 100$, where $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} - \hat{k}$, then $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to:
- (A) 27 (B) 14 (C) -27 (D) -14

Space for Rought Work

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

1. A point moves such that sum of squares of its distances from points ; (1, 1), (1, -1), (-1, 1), (-1, -1) is 36 units. Locus of this moving point is a circle of radius 'R'. Then $R^2 =$ _____.
2. The smallest natural number n , such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is _____.
3. $z = \frac{1+i\sqrt{3}}{2}, i = \sqrt{-1}$. The value of $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \dots + \left(z^{27} + \frac{1}{z^{27}}\right)^2 - 50$ is:
4. If the length of the perpendicular from the point $(\beta, 0, \beta)$ ($\beta \neq 0$) to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then $\beta + 5$ is equal to _____.
5. The total number of integral values of λ ($\lambda \neq 0$) for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{1}{\lambda}$ in 'x' has no real roots, is _____.

Space for Rought Work

6. If $y = y(x)$ is a function of 'x' such that $2x + y = e^{2xy}$, then $\frac{d^2y}{dx^2}$ at $x = 0$ is equal to _____.

7. Let $a, b \in R, b \neq 0$. Define a function

$$f(x) = \begin{cases} a \sin\left(\frac{\pi}{3}5x - \frac{3}{2}\right), & \text{for } x \geq 0 \\ \frac{\tan 3x - 2 \sin 3x}{bx}, & \text{for } x < 0 \end{cases}$$

If $f(x)$ is continuous at $x = 0$, then $6 - ab$ is equal to _____.

8. The area of region $S = \{(x, y) : x^2 < y < 2 - x\}$ is $\frac{\lambda}{2}$, then λ is equal to _____.

9. A wire of length $60m$ is cut into two pieces one of the pieces is bent to form a circle and other is bent to form a square. If the sum of area of two figures is minimum and the circumference of circle is ' λ ', then $\left(\frac{2}{15\pi} + \frac{1}{30}\right)\lambda =$ _____.

10. The number of three-digit odd numbers, formed by the digits 0, 2, 4, 5, 6, 7, 9 if repetition of digits is not allowed is $15K$, then $K =$ _____.

Space for Rought Work